TRINITY COLLEGE

Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS
METHODS
UNIT 3,4

Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words _____

_____

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.
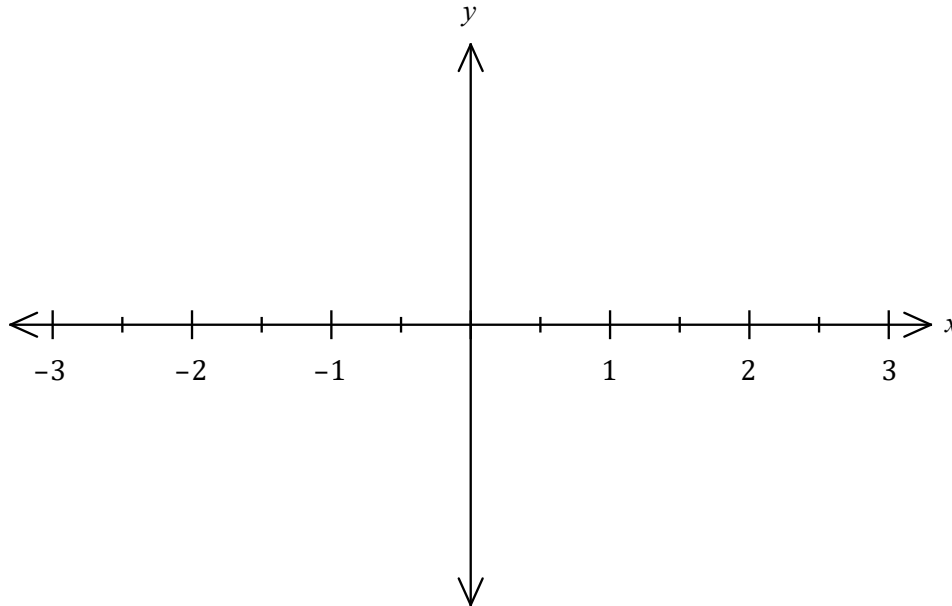
Working time: 100 minutes.

Question 9 **(8 marks)**

Let $f(x) = 2x^4 + ax^2 + 1$.

(a) Sketch the graph of $y = f(x)$ when $a = -16$. (4 marks)

(b) Show that the graph of $y = f(x)$ will always have a maximum turning point at $x = 0$
     if $a < 0$. (4 marks)

Question 10 **(6 marks)**

(a) List A contains the digits in the first 200 decimal places of π. The relative frequencies of the digits are:

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	0.095	0.100	0.120	0.095	0.110	0.100	0.080	0.060	0.125	0.115

Determine the probability that a randomly selected digit from list A

(i) is odd. (1 mark)

(ii) is a factor of 18, given that it is not odd. (2 marks)

(b) The discrete random variable X is defined by

$$P(X = x) = \begin{cases} \frac{1}{6} & x = 0, 1, 2, 3, 4, 5 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the expected value and variance of X. (3 marks)

Question 11 **(7 marks)**

A hot potato was removed from an oven and placed on a cooling rack. Its temperature T, in degrees Celsius, t minutes after being removed from the oven was modelled by

$$T = 23 + 187e^{kt}.$$

The temperature of the potato at $t = 8$ is half the initial temperature.

(a) Determine the value of the constant k. (3 marks)

(b) The temperature of the potato eventually reached a steady state. Determine the time taken for its temperature to first fall to within $1\,°C$ of this steady state. (2 marks)

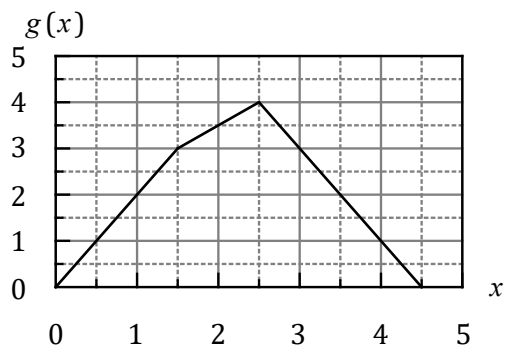(c) Determine the time at which the potato was cooling at a rate of $1\,°C$ per minute. (2 marks)

See next page

Question 12 **(8 marks)**

The graph of function g, and a table of values for function f and its derivatives are shown below.

x	1	2	3
$f(x)$	2	1	3
$f'(x)$	3	2	2
$f''(x)$	-1	-2	1

$g(x)$

(a)   Evaluate $h'(k)$ when

(i) $h(x) = g\big(f(x)\big)$ and $k = 3$. (3 marks)

(ii) $h(x) = f(x) \div g(x)$ and $k = 1$. (3 marks)

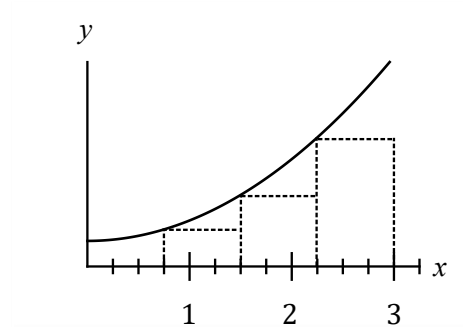(b)   Evaluate $h''(2)$ when $h'(x) = f'(x) \times g'(x)$. (2 marks)

Question 13 (8 marks)

The graph of $y = f(x)$ is shown at right with 3 equal width inscribed rectangles. An estimate for the area under the curve between $x = 0.75$ and $x = 3$ is required.

The function f is defined as $f(x) = 4x^2 + 5$ and let the area sum of the 3 rectangles be S_3.

(a) State the height and width of the second rectangle. (1 mark)

(b) Calculate the value of S_3. (3 marks)

(c) Explain, with reasons, how the value of width of the rectangle and the area estimate S_n will change as the number of inscribed rectangles increases. (2 marks)

(d) Determine the limiting value of S_n as $n \to \infty$. (2 marks)

See next page

Question 14 **(8 marks)**

The area A of a regular polygon with n sides of length x is given by

$$A = \frac{n\,x^2 \cos\left(\frac{\pi}{n}\right)}{4\sin\left(\frac{\pi}{n}\right)}$$

(a) Simplify the above formula when $n = 6$ to obtain a function for the area of a regular hexagon. (2 marks)

(b) Use the increments formula to estimate the change in area of a regular hexagon when its side length increases from 10 cm to 10.5 cm. (3 marks)

(c) Use the increments formula to estimate the change in area of a regular polygon with sides of length 10 cm when its number of sides increases from 29 to 31. (3 marks)
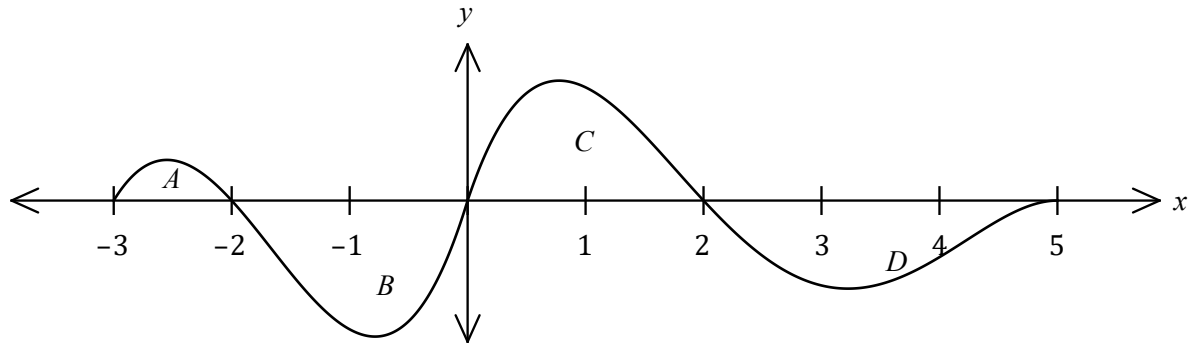
Question 15 **(8 marks)**

(a) It is known that 17% of a large number of smoke alarms in a complex of buildings are faulty. If an electrician randomly selects 8 alarms for inspection, determine

    (i) the probability that none of the alarms will be faulty. (2 marks)

    (ii) the probability that more than three alarms are faulty, given that at least one is faulty. (2 marks)

    (iii) the standard deviation of the distribution of the number of faulty alarms. (1 mark)

(b) In a newer complex that also has a large number of smoke alarms, only 7% are faulty. Determine, with reasoning, the minimum number of alarms that should be inspected so that the probability that at least one of them will be faulty is more than 99%.

(3 marks)

See next page

Question 16 **(7 marks)**

Regions A, B, C and D bounded by the curve $y = f(x)$ and the x-axis are shown on this graph:

The areas of A, B, C and D are $9, 30, 23$ and 21 square units respectively.

(a) Determine the value of

(i) $\displaystyle\int_0^5 f(x)\,dx$. (1 mark)

(ii) $\displaystyle\int_{-3}^2 5f(x)\,dx$. (2 marks)

(iii) $\displaystyle\int_{-2}^5 (f(x) - 4)\,dx$. (2 marks)

(b) Explain why $\displaystyle\int_{-2}^2 f'(x)\,dx = 0$. (2 marks)

Question 17   **(8 marks)**

The volume, V litres, of fuel in a tank is reduced between $t = 0$ and $t = 48$ minutes so that

$$\frac{dV}{dt} = -175\pi \sin\left(\frac{\pi t}{48}\right)$$

(a)   Determine, to the nearest litre, the amount of fuel emptied from the tank

         (i)   in the first minute.   (3 marks)

         (ii)   in the last 7 minutes.   (1 mark)

The tank initially held 18 600 litres of fuel.

(b)   Determine the volume of fuel in the tank 5 minutes after the volume in the tank reached 12 000 litres.   (4 marks)

Question 18   **(8 marks)**

When an electronic device is run, it randomly generates one of the first four triangle numbers. The discrete random variable X is the number generated in one run of the device and the table below shows its probability distribution.

x	1	3	6	10
$P(X = x)$	a	b	0.2	0.3

E(X) = 5.6.

(a)   Determine the value of the constant a and the value of the constant b.   (3 marks)

(b)   The electronic device is run 3 times. Determine the probability that

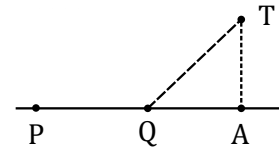   (i)   the number 6 will be generated exactly twice.   (2 marks)

    (ii)   the sum of the numbers generated is at least 23.   (3 marks)

Question 19 (8 marks)

An offshore wind turbine T lies 9 km away from the nearest point A on a straight coast. It must be connected to a power storage facility P that lies on the coast 40 km away from A.

Engineers will lay the cable in two straight sections, from T to Q, where Q is a point on the coast x km from A, and then from Q to P.

The cost of installing cable along the coastline is $4000 per km and offshore is $5000 per km.

(a) Determine, to the nearest hundred dollars, the cost of installing the cable when Q lies midway from A to P. (2 marks)

(b) Show that C, the cost in thousands of dollars, to run the cable from T to Q to P, is given by $C = 5\sqrt{x^2 + 81} - 4x + 160$. (2 marks)

(c) Use calculus techniques to determine, with justification, the minimum cost of laying the cable from T to P. (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 20   **(8 marks)**

Small body P moves in a straight line with acceleration a cm/s^2 at time t s given by

$$a = At + B$$

Initially, P has a displacement of 8 cm relative to a fixed point O and is moving with a velocity of 4 cm/s. Three seconds later, P has a displacement of 3.8 cm and a velocity of -5.9 cm/s.

(a)   Determine the value of the constant A and the value of the constant B.   (6 marks)

(b)   Determine the minimum velocity of P.   (2 marks)

Question 21   **(5 marks)**

(a)   Determine the value of the constant a and the value of the constant b that make each of the following statements true, given that $f(x)$ is a polynomial:

(i) $\displaystyle \int_{2}^{3} f(x)\,dx + \int_{-1}^{2} f(x)\,dx = \int_{a}^{b} f(x)\,dx.$ (1 mark)

(ii) $\displaystyle \int_{-3}^{2} f(x)\,dx + \int_{2}^{a} f(x)\,dx - \int_{-3}^{b} f(x)\,dx = \int_{2}^{4} f(x)\,dx.$ (2 marks)

(b)   Given $\displaystyle F(x) = \int_{a}^{x} (2t - 3)\,dt$, determine the value of x that minimises $F(x)$ and justify it is a

minimum.   (2 marks)

End of questions   S

Supplementary page

Question number: _____

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